QP CODE: 26800171

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Reg No : .....

Name : .....

INTEGRATED M.Sc. DEGREE EXAMINATION, DECEMBER 2025

First Semester

INTEGRATED M.Sc. BASIC SCIENCE-PHYSICS

COMPLEMENTARY - IPH1CM05 - MATHEMATICS - I DIFFERENTIAL CALCULUS, MATRICES AND TRIGONOMETRY

2021 Admission Onwards

221F5FCE

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Use implicit differentiation to find $\frac{dy}{dx}$ if $x = \tan y$.
2. Find the slope of the curve $x^3 + xy - y^2 = 1$ through the point $(2, 3)$.
3. Find the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(4, -5)$ if $f(x, y) = x^2 + 3xy + y - 1$.
4. If $z = \ln \sqrt{x^2 + y^2}$, prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.
5. Find the derivative of $w = xy$ with respect to t at $t = \frac{\pi}{2}$ along with $x = cost, y = sint$.
6. Define characteristic equation and characteristic roots of a polynomial.
7. Define characteristic space.
8. If x and y are real or complex numbers, prove that $\sin(x + y) = \sin x \, cosy + \cos x \, siny$.
9. If x is real, show that $\cosh^{-1} x = log[x + \sqrt{x^2 - 1}]$.
10. Separate into real and imaginary parts the expression $\sinh(x + iy)$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Find the extreme values of the function $y = x^3 + x^2 - 8x + 5$ and where they occur.
12. Find $\lim_{x \to \infty} (ln\ x)^{\frac{1}{x}}$.
13. Sketch the domain of $f(x, y) = \sqrt{y - x^2}$.Find boundary of the functions domain. Determine if the domain is an open region, a closed region or neither.

14. Using chain rule express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ in terms of r and θ if $w = \tan^{-1}(\frac{y}{x})$, $x = r\cos\theta, y = r\sin\theta$. Also evaluate $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ at the point $(1, \frac{\pi}{6})$.

15. Show that the system of equations $2x + 6y + 11 = 0$ are not consistent.
$$6x + 20y - 6z + 3 = 0$$
$$6y - 18z + 1 = 0$$

16. Solve $x + 2y + 3z = 0$
$$2x + y + 3z = 0$$
$$3x + 2y + z = 0$$

17. Express $\cos 7\theta$ in terms of $\cos\theta$.

18. Sum the series $\cos\alpha + {}^nC_1\cos(\alpha+\beta) + {}^nC_2\cos(\alpha+2\beta) + \ldots + \cos(\alpha+n\beta)$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a. Draw the tree diagram for the function $z = f(u,v,w)$, $u = g(t)$, $v = h(t)$, $w = k(t)$ and also write the chain rule.
b. Use chain rule to evaluate $\frac{dw}{dt}$ at $t = 1$ if $w = \ln(x^2 + y^2 + z^2)$, $x = \cos t$, $y = \sin t$, $z = 4\sqrt{t}$..
c. Evaluate $\frac{dw}{dt}$ at $t = 0$ if$w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

20. Find the rank of a) $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 1 & 2 \end{bmatrix}$.

21. Using Cayley Hamilton theorem , show that $A^3 - 6A^2 + 11A - 6I = 0$ and also find $A^{-2}$where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$.

22. a. Expand $\cos^5\theta\sin^7\theta$ in a series of sines of multiples of θ.
b. Prove that $\sin^8\theta = \frac{1}{128}[\cos 8\theta - 8\cos 6\theta + 28\cos 4\theta - 56\cos 2\theta + 35]$.

(2×5=10 weightage)